Assignment 10

1. Use the finite-difference method with $h = 0.2$ to approximate a solution to the boundary value problem

$$\begin{aligned} u^{(2)}(x) &= 0.3u^{(1)}(x) + 0.1u(x) - x - 0.2 \\ u(0) &= 2 \\ u(1) &= 3 \end{aligned}$$

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3. Given the function $u(\mathbf{x}, t) = t\, x_1 x_2 - x_1 + 2x_2$, approximate the partial derivative with respect to time and the gradient at the point $t = 0.2$ and $\mathbf{x} = \begin{pmatrix} 0.3 \\ -0.5 \end{pmatrix}$ using a value of $\Delta t = h = 0.1$.

4. Given the function in Question 3, approximate all three second partials: with respect to t, x_1, and x_2.

5. In class, we did not discuss an explicit formula for $\frac{\partial^2}{\partial x \partial y} u(x, y)$. Come up with such a formula. Show that your formula works by calculating this second partial explicitly using the process shown in your calculus course for the function in $xe^x\sin(y)$ at $x = 1$ and $y = 2$, and then calculating your approximation using $h = 0.01$.

6. Demonstrate that the formula for $\frac{\partial^2}{\partial x \partial y} u(x, y)$ that you found in Question 5 is the same formula you would find if you were to approximate $\frac{\partial^2}{\partial y \partial x} u(x, y)$.

7. Approximate a solution to the heat equation with four steps in time if the boundary conditions are $u_a(t) = 0$ and $u_b(t) = 2$ and the initial state is $u_0(x) = 1 - x$ if the interval in space is $[0, 1]$ and $h = 0.2$. The coefficient $\alpha = 4$. You should use a Δt, as described in the course notes, to ensure convergence.